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Detection of Multiple Upper Outliers in Exponential Sample under Slippage Alternative

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Abstract: This paper aims to compare the empirical powers of some statistics for detecting multiple upper outliers in exponential samples under slippage alternative. In addition to that we also investigate masking effect for various degree of discordancy parameter. The results which are based on simulation study, indicate that the maximum likelihood ratio test statistic is better than the other statistics followed by Dixon type test statistics to deal with upper outliers in exponential samples. Also in case of combating masking the maximum likelihood ratio test statistic is better and the test proposed by Lalitha and Kumar (2012) precede Dixon type test.

Keywords: Outlier, slippage alternatives, masking effect, discordancy parameter.

I. INTRODUCTION

Exponential distribution has many applications in life-testing experiments and reliability engineering. It is used to model the behavior of units that have a constant failure rate. This distribution describes the time between events in a Poisson process. With the location parameter μ and scale parameter θ, the density of the random sample $X_1, X_2, \ldots, X_n$ from an exponential distribution defined by

$$f(x;\ \mu,\theta) = \frac{1}{\theta} exp\left[-\left(\frac{x-\mu}{\theta}\right)\right] \qquad x > \mu > 0, \theta > 0 \qquad (1)$$

Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the corresponding ordered sample. We are interested in identifying the suspected $k\ (< n)$ observations. An outlier is an observation that deviates so much from other observations as to arouse suspicion that it was generated by a different mechanism (Hawkins, 1980). Outliers are unusual observations, but not necessarily errors. In the beginning outlier detection work was concentrated on normal distribution but later on other distributions are also drawing attention from the researchers. Testing for single outlier in exponential samples had been studied by various authors including Laurent (1963), Basu(1965), Likes(1966), Mount and Kale (1973), Lewis and Filler (1979), Kabe(1970), etc. Owing to the masking problem, there are lots of work on multiple outlier test including Chikkagoudar and Kunchur (1983), Zerbet and Nikulin (2003), Shadrokh and Pazira (2010), Lalitha and Kumar (2012), Kumar (2013), Lin and Balakrishnan (2014), etc. There are different types of alternative hypothesis under which the performance can be measured for different test. Here we are concerned with the labelled slippage hypothesis. The slippage alternative is that "while the majority of the observations are identical to some common distribution $f(.)$, some small number of them have large probabilities in the tail regions of $f(.)$, and are said to have *slipped*" (Hawkins, 1980). Here we are interested to evaluate the powers of a few well referred block test in detecting multiple outliers under slippage alternative. In addition to that we also investigate the masking effect of the block tests. Here the outliers are supposed to have generated due to the scale slipped in the sample. The rest of the paper is organized as follows: some earlier works are highlighted in section II. In Section III, the test statistics to be compared are briefly described. The performance criteria to evaluate the empirical power and the masking effect are given in Section IV and V respectively. The simulation study are given in Section VI and it results are followed in Sections VII, VIII and IX. Finally concluding remarks are made in Section X.

II. EARLIER WORKS IN OUTLIER DETECTION FOR UNIVARIATE EXPONENTIAL SAMPLES

Until the 1960s, most of the published work on outliers in univariate samples were in the context of normal distributions (Barnett and Lewis, 1994). Laurent (1963) derived the distribution of test stastistic $\frac{(\bar{X}-X_{(1)})}{(X_{(n)}-X_{(1)})}$ for exponential samples. Basu (1965) applied this statistic in the rejection of a single observation in an exponential sample. Likes (1966) modified Laurent's work in the derivation of Dixon's ratio-of-gaps statistics for exponential distributions. Kabe (1970) gave an algorithm for the efficient computation of Likes's critical values and showed how the distributions obtained by Likes can be expressed in terms of finite series of beta functions and the probability of rejection can be calculated on a desk calculator. Kimber (1979) proposed test for a single outlier in a gamma sample with unknown shape and scale parameter. Kimber and Stevens (1981) derived the null distribution of the maximum likelihood ratio test statistic for the two upper outliers in an exponential sample and also they provided a simple inequality for the significance probability. They used sensitivity contours to prove the efficiency. Later on Kimber (1982) proposed a sequential procedure for testing up to k upper outliers in an exponential sample. Chikkagoudar and Kunchur (1983) derived the distributions of two types of statistics in exponential samples.

The two statistics are (i) based on the ratio of sum of suspected observations to the sum of sample observations, and (ii) Dixon's type statistic.

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For $k = 1$ and $k = 2$ the null distribution of both statistics corresponds to that of Likes (1966).

Bendre and Kale (1985) made power comparison between the Modified Dixon tests and Cochran test for exponential models and they showed that in presence of two outliers these methods remain free from masking effects.

Zhang (1998) extends the null distribution of the maximum likelihood ratio test statistic and corresponding percentage points up to $n \leq 100$ and $k \leq 8$, which was originally confined to $k = 2$ given by Fisher(1929) and by Kimber and Stevens(1981) for k upper outliers and by Lewis and Fieller (1979) for k lower outliers. Also Zhang (1998) found a way of determining k, which can reduce the masking or swamping' effects.

Zerbet and Nikulin (2003) proposed a new statistic for detecting outliers in exponential case. Through a power comparison they showed that their statistic performs better that Dixon's statistics. They also showed that in case of only one upper outlier (i.e., with $b = 1$ and $k = 1$) the distribution becomes same that of Likes statistic (1966). Shadrokh and Pazira (2010) proposed another statistic under slippage alternative following the methodology of Zerbt and Nikulin(2003). They showed that their newly proposed statistics is more powerful than Dixon's statistic. Lalitha and Kumar (2012) proposed a statistic belonging to "gap-test" family. It is based on all observations for testing upper outlier(s) with a slippage alternative in an exponential sample. In a concluding remark, Lalitha and Kumar said that their statistic performs better than the maximum likelihood ratio test and Zerbet and Nikulin (2003) test statistic for any n and k in respect of power. They preferred it as does not required any table for detection of outliers.

All the above mentioned statistics were derived with known origin. For multiple outliers with slippage alternative in an exponential sample when the origin is unknown, Kumar (2013) suggested a procedure based on a ratio of two maximum likelihood estimates. And coincidently the same statistic was suggested by Balasooriya and Gadag (1994) based on score function. Kumar (2013) developed a general procedure to construct test statistic for outlier detection in one-parameter exponential family. The procedure consists in finding the ratio of the two maximum likelihood estimate (MLE) of the scale parameter. One of the MLEs is obtained from the complete data log-likelihood and another from its conditional expectation given the expected observations. He proposed a general framework for one-parameter exponential sample with density $f(x|\theta) = h(\theta)d(x)exp\{-\alpha(\theta)v(x)\},\ a < x < b$ with scale parameter θ, $h(.)$; $\alpha(.)$; $d(.)$ and $v(.)$ are known functions and $h(.)$; positive valued. To derive the MLE of θ the following normal equation was proposed:

$$\frac{h'(\theta)}{h(\theta)\alpha'(\theta)} = \frac{\sum_{i=1}^{n} v(x_{(i)})}{n}$$

Then assuming that there are $k_1$lower outliers and k_2 upper outliers and using the information on the remaining $n - k_1 - k_2$ observations the MLE of θ is to be derived from the following normal equation:

$$\frac{h'(\theta^{(m+1)})}{h(\theta^{(m+1)})\alpha'(\theta^{(m+1)})}$$

$$= \frac{1}{n}\left[\sum_{i=k_1}^{n-k_2} v(x_{(i)}) + E_{\theta^{(m)}}\left(v(x_{(i)})\middle|x_{(k_1+1)}\right) + E_{\theta^{(m)}}\left(v(x_{(i)})\middle|x_{(n-k_2)}\right)\right]$$

where m is the number of iterations. The ratio of the two estimates will be the required test statistic.

Lin and Balakrishnan (2014) proposed an algorithm for evaluating the null joint distribution of Dixon-type test statistics for testing discordancy of k upper outliers in exponential samples by applying the recursion of Huffer repeatedly.

III. TEST STATISTICS TO BE COMPARED

Our aim of this paper is to compare the empirical powers of some test in detecting multiple upper outliers in exponential samples under slippage alternative. Consider a random sample $X_1, X_2, \dots, X_n$ ideally from an exponential distribution defined in (1). However out of n observations some unknown set of k observations are suspected to have come from a different exponential distribution $f(x|\theta/b), 0 < b < 1$, whereas the remaining $(n - k)$ observations, which form the main part of the sample are from the distribution $f(x|\theta)$. These k observations in the sample coming from a distribution with greater mean are called upper outliers (Chikkagoudar and Kunchur, 1983). Here the aim is that of testing the hypothesis

H_0: $X_1, X_2, \dots, X_n$ are from the distribution $f(x|\theta)$, against the alternative

H_k: $X_{i1}, X_{i2}, \dots, X_{ik}$ are from the distribution $f(x|\theta/\beta), 0 < \beta < 1$, and $(i_1, i_2, \dots, i_k)$ is an unknown subset of $1,2, \dots, n$.

Following are the five test statistics, considered for power comparison.

Dixon type statistic: Basu (1965) proposed a Dixon type test statistic, which can be written as

$$D_k = \frac{(X_{(n)} - X_{(n-k)})}{X_{(n)}} \qquad (2)$$

its null distribution was obtained by Likes (1966) and Kabe (1970), Chikkagoudar and Kunchur (1983). The large value of D_k indicates the presence of upper outliers (Kumar, 2013).

The maximum likelihood ratio test statistic: This statistic is based on the ratio of the sum of observations suspected to be outliers to the sum of all observations in the sample. Denoted by T_k, it is given as:

$$T_k = \frac{(X_{(n-k+1)} + \cdots + X_{(n)})}{\sum_{i=1}^{n} X_{(i)}} \qquad (3)$$

The null distribution was derived by Chikkagouder and Kunchur (1983), Zhang (1998), Lin and Balakrishnan (2014). A larger value of T_k above a specified level

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indicates the preence of k upper outliers. Here, we have used the critical values for T_k from the tables given by Zhang (1998).

Zerbet and Nikulin test statistic: Zerbet and Nikulin (2003) proposed a new statistic for detecting outliers in exponential case. They used the idea of Chauvenet and using characteristic function approach they derived the distribution of the test statistic, which is defined as

$$Z_k = \frac{X_{(n-k)} - X_{(1)}}{\sum_{j=n-k+1}^{n}\left(X_{(j)} - X_{(1)}\right)} \tag{4}$$

Lalitha and Kumar (2012) mentioned that a smaller value of Z_k indicates the presence of k upper outliers in the sample. But there is some discrepancy in the tabulated critical values which are proved to be incorrect. Here we have recalculated the critical values through simulation and is given in the end of the paper.

Shadrokh and Pazira test statistic: Adopting the same methodology as Z_k , Shadrokh and Pazira (2010) proposed a test statistic for multiple outliers and it may be written as

$$SP_k = \frac{X_{(n)} - X_{(n-k)}}{\sum_{j=n-k+1}^{n}(X_{(n)} - X_{(j)})} \tag{5}.$$

they obtained the distribution of T_k and through a power comparison they also showed that T_k is more powerful than Dixon's statistic.

Lalitha and Kumar test statistic: For testing upper outlier(s) with a slippage alternative in an exponential sample, Lalitha and Kumar (2012) proposed a statistic based on all observations. For a single upper outlier it is given as:

$$LK_1 = \frac{X_{(n)} - X_{(n-1)}}{S_n} \tag{6}$$

where $S_n = \sum_{j=1}^{n} x_{(j)}$. This test is a member of "gap-test" family with denominator $\sum_{j=1}^{n} x_{(j)}$ for the gap $x_{(n)} - x_{(n-1)}$. They obtained the null distribution of LK_k as: $\{T(X) > A_{n,\alpha,1}|H_0\} = \{1 - A_{n,\alpha,1}\}^{n-1}$. The critical values can be obtain from $A_{n,\alpha,1} = (1-\alpha)^{1/(n-1)}$ directly. For many outliers, say for k upper outliers the extension is:

$$LK_k = \frac{X_{(n)} - X_{(n-k)}}{S_n}\ ; k \geq 1 \tag{7}$$

A larger value of LK_k will indicate the presence of k upper outliers in the sample. The exact null distribution of LK_k is intricate for $k \geq 2$. The approximate critical values of $A_{n,\alpha,k}$ may be obtained by

$$1 - \sum_{j=1}^{k}\left\{\frac{j-1+\alpha}{j}\right\}^{1/(n-1)}$$

where is the estimated critical value at α level of significance.

IV. PERFORMANCE CRITERIA OF TESTS

To evaluate the efficiencies of the tests we have the option of comparing the test at the same level of significance depending on the alternative hypothesis. Suppose $T(\boldsymbol{X})$ be the value of test statistics, whose larger value indicates the outlyingness of the observations and $A_{n,\alpha}$ the critical value at the preassigned significance level α such that $P\{T(\boldsymbol{X}) > A_{n,\alpha}|H\} = \alpha$. Then $x_{(n-k+1)},\ldots,x_{(n)}$ are declared outliers if $T(\boldsymbol{X}) > A_{n,\alpha}$. As we are assuming slippage alternative, in this context David (1970) suggested five probabilities as reasonable measure of the performance of $T(X)$, one of which is: P=$P\{T(X) > A_{n,\alpha}|\bar{H}\}$

This is the probability under $\bar{H}$ that the outlier is identified as discordant, in other words the power function.

V. TESTING FOR MASKING EFFECT

Masking effect is the inability of a testing procedure to identify even a single outlier in the presence of a several suspected values (Tietjen and Moore, 1972). The masking effect in cases of tests for outlier(s) is defined and quantified by the loss in power due to the presence of more than the anticipated number of outliers in the sample (Bendre and Kale, 1985). Under the labeled slippage alternative H_k, $X_{(i)}/X_{(n)} \xrightarrow{P} 0$ as $\beta \to 0$, $i = 1,2,\ldots,n-k$. Assuming $T(\boldsymbol{X})$ as a test statistic to detect a single outlier, Bendre and Kale (1985) used some measures to define and quantify the masking effect. They defined $P_k(\beta) = P\{T(\boldsymbol{X}) \in A_{n,\alpha}|H_k\}$ as the power of the test, and $P_c(\beta) = P\{T(\boldsymbol{X}) \in A_{n,\alpha}|H_c\}$ as the power the test under an alternative different from one specified by H_k. As a uniform test it is assumed that $P_c(\beta)$ will monotonically converge to unity with increase in the degree of discordancy. Now for $> k$, the same test would have less power under the $H_{c>k}$. Thus $P_c(\beta)$ may be less than the measure $P_k(\beta)$, and it may reduce to zero. The masking effect is then measured by the quantity:

$$M_b = P_k(\beta) - P_c(\beta) \tag{8}$$

Under extreme slippage, the limiting masking effect $M = \log_{\beta\to\beta_0} M_\beta$; where β_0 is the limiting discordancy value. A test is said to be prone to masking effect if the measure M is positive, and it is free from masking if $M = 0$.

VI. SIMULATION STUDY AND RESULTS

As stated in the previous Section, we are using power function P=$P\{T(X) > A_{n,\alpha}|\bar{H}\}$. Here the powers are evaluated in terms of the proportion of rejection. For this we are using Monte Carlo Simulation Techniques and the exponential samples were generated using $-\theta\ln(1-R_U)$; where $R_U(0,1)$ is uniform deviate and WLOG the scale parameter θ has been assumed as unity . For each generated sample the value of the statistics are calculated and these are compared with the critical value to accept or reject the hypothesis. If it is rejected (i.e., outlier detected), it is counted and the process get repeated.

We have repeated 10,000 times for each sample of size $n = 10$. The proportion of rejection is being displayed in the following table and graphs.

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Table-1: Empirical power for exponential samples for $n = 10, k = 2$, *at* $\alpha = 5\%$ *and* 1%

β	Level α	LK LK_k	Dixon's D_k	MLR T_k	SP(new) SP_k	ZN(new) Z_k
1.0	0.05	0.0475	0.0475	0.0489	0.05	0.05
	0.01	0.0081	0.0091	0.0093	0.01	0.0101
0.9	0.05	0.0487	0.0466	0.0494	0.0519	0.0496
	0.01	0.0084	0.0099	0.01	0.0098	0.0101
0.8	0.05	0.0538	0.0521	0.0522	0.0497	0.0478
	0.01	0.0105	0.0114	0.0114	0.0096	0.0095
0.7	0.05	0.0571	0.0552	0.0605	0.0489	0.0477
	0.01	0.0122	0.0123	0.0143	0.0092	0.0095
0.6	0.05	0.0727	0.069	0.07	0.0489	0.0436
	0.01	0.0184	0.0184	0.0186	0.0096	0.007
0.50	0.05	0.0939	0.0861	0.0889	0.0466	0.0418
	0.01	0.0269	0.0264	0.0255	0.0074	0.0062
0.40	0.05	0.1325	0.1229	0.1271	0.0435	0.0384
	0.01	0.0444	0.0441	0.0431	0.0085	0.0031
0.3	0.05	0.1767	0.1878	0.172	0.0435	0.0328
	0.01	0.0685	0.0772	0.0682	0.0087	0.001
0.20	0.05	0.3485	0.3292	0.3547	0.0396	0.0189
	0.01	0.177	0.1694	0.188	0.0064	0.0003
0.10	0.05	0.6267	0.6086	0.6209	0.0409	0.0069
	0.01	0.37	0.4244	0.4351	0.0081	0.000
0.09	0.05	0.6619	0.6489	0.6962	0.0405	0.0062
	0.01	0.4399	0.4705	0.5319	0.0078	0.000
0.07	0.05	0.7624	0.7359	0.7933	0.0413	0.0042
	0.01	0.5375	0.5747	0.6618	0.0084	0.000
0.05	0.05	0.8384	0.8322	0.8552	0.0444	0.003
	0.01	0.6401	0.7042	0.7454	0.0087	0.000
0.03	0.05	0.9239	0.9223	0.9493	0.0469	0.0005
	0.01	0.798	0.8496	0.9017	0.0094	0.000
0.01	0.05	0.9896	0.9887	0.9935	0.0496	0.0001
	0.01	0.9561	0.9754	0.9844	0.0099	0.000

Table-2: Values of $P_k(\beta)$ for different values of n and k at $\alpha = 0.05$

Sample size	No. of outliers	β (discordancy parameter)								
		LK_k			D_k			T_k		
N	k	0.9	0.5	0.1	0.9	0.5	0.1	0.9	0.5	0.1
10	2	0.0487	0.0939	0.6267	0.054	0.0861	0.6086	0.0494	0.0889	0.6606
10	3	0.0263	0.0433	0.1679	0.0225	0.0334	0.1272	0.0339	0.0564	0.2947
20	2	0.0515	0.0965	0.691	0.048	0.0871	0.6487	0.0494	0.0926	0.7091
20	3	0.0407	0.0755	0.4184	0.0359	0.0588	0.3281	0.0438	0.0843	0.5292
30	2	0.0493	0.0877	0.6918	0.0585	0.0956	0.6707	0.0478	0.0865	0.7134
30	3	0.0419	0.0810	0.5205	0.0491	0.0797	0.4462	0.0442	0.0879	0.6067

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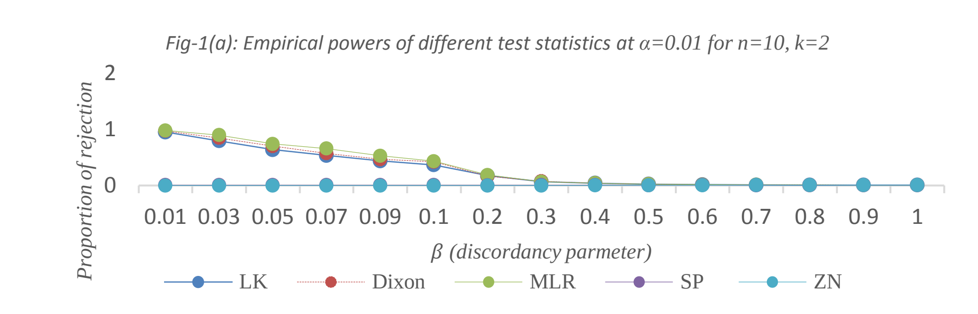

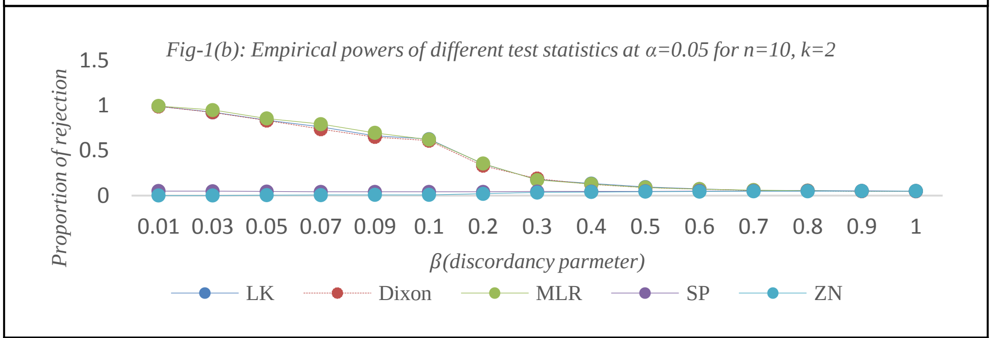

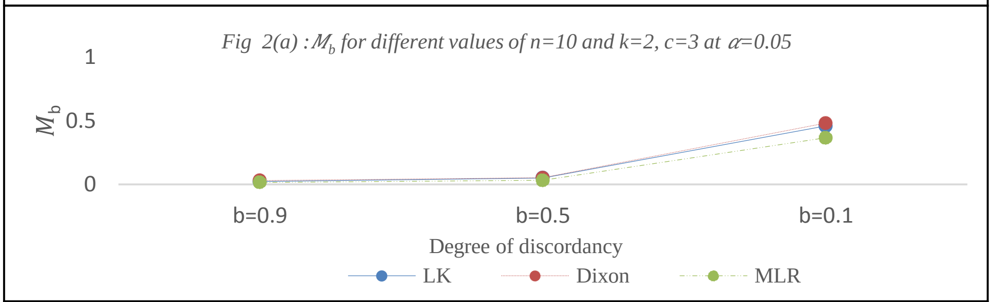

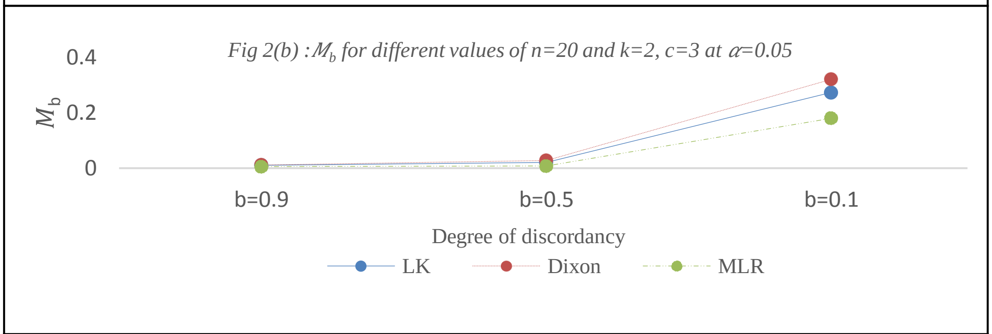

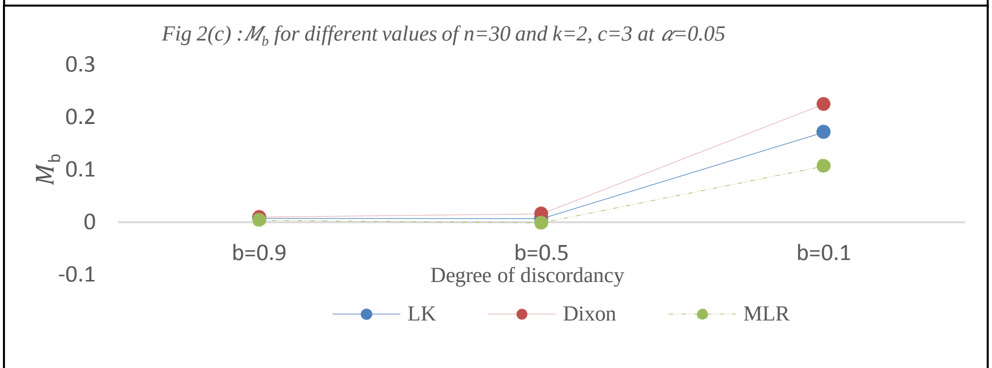

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VII. CRITICAL VALUES OF TWO NEW TEST STATISTICS

Before applying the above measure to evaluate empirical power, we had checked the level under null hypothesis. We found that the critical values of two test statistics viz., SP_k and Z_k do not satisfy the criteria under null hypothesis. So we computed the critical values afresh. Table-1 shows the discrepancy observed in the empirical power after using the original and newly computed critical values. Here Z_k may be considered to satisfy the level at 5 percent but for 1 percent it is far from conformity. While those of SP_k are abruptly different from the desired level. For user convenience we are providing the newly computed critical values in the appendix.

Table-3: Discrepancy observed for the critical values

	Level α	SP(new) SP_k	ZN(new) Z_k	SP	ZN
1.0	0.05	0.05	0.05	1.00	0.049
	0.01	0.01	0.0101	1.00	0.1045
0.8	0.05	0.0497	0.0478	1.00	0.0478
	0.01	0.0096	0.0095	1.00	0.1014
0.6	0.05	0.0489	0.0436	1.00	0.0428
	0.01	0.0096	0.007	1.00	0.0954
0.40	0.05	0.0435	0.0384	1.00	0.0414
	0.01	0.0085	0.0031	1.00	0.0874
0.20	0.05	0.0396	0.0189	1.00	0.038
	0.01	0.0064	0.0003	1.00	0.0823

VIII. EMPIRICAL POWERS

From the table-1 and figures 1(a) and 1(b) we observed that out of the five test, SP_k and Z_k have negligible powers. Only for small shift in the scale all have more or less same potentiality to detect multiple outliers. At 5 percent level SP_k has the highest power for $\beta = 0.9$ followed by Z_k, but for $\beta < 0.9$ both of them loses their power drastically. Rest of the three viz., LK_k, D_k and T_k appeared to be equivalent for $\beta \geq 1.5$ at both level of significance.

For larger slippage in the scale $(\beta < 0.15)$, T_k shows highest power. And as expected from an uniform powerful test the power of LK_k, D_k and T_k monotonically converge to unity with increase in the degree of discordancy, i.e., $P_k(\beta) \to 1$ as $\beta \to 0$. Also at 1 percent level the power discrepancy is much lower than 5 percent level.

IX. COMPARISON OF MASKING EFFECT OF LK_k, D_k AND T_k

The following table-3 shows the loss in power due to masking. As stated earlier and evidenced by (8), that if the measure M is positive then a test is said to be prone to masking effect, and it is free from masking if $M = 0$. From the table and graphs it is clear that all the three test are affected by masking. For small slippage all the three tests are equivalent and the masking effects are very negligible.

The test T_k is again the best followed by LK_k and D_k. As the degree of discordancy increases and the sample size increases the amount masking effect also become noticeable. But at some point of discordancy $\beta = (0.5)$ there is an unexpected situation regarding the test T_k.

X. CONCLUDING REMARKS

From the results we can comment that out of the five test statistics considered here, only three of them are capable of detecting upper outliers under scale slippage. Those are test based on maximum likelihood ratio test, test of Dixon type and test proposed by Lalitha and Kumar. In case of masking, maximum likelihood ratio (MLR) test does not exhibit the uniformity for all values of discordancy.

However for large difference between c and k this uniformity is maintained. But above all, when there is large slippage in scale for an exponential sample the test based on MLR is better than others.

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Appendix:

n	k =1			k =2			k =3			k =4		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
6	0.9223	0.9601	0.9925	0.3805	0.4122	0.4587	0.1941	0.2220	0.2648			
7	0.9325	0.9660	0.9933	0.3950	0.4266	0.4679	0.2169	0.2415	0.2792			
8	0.9360	0.9673	0.9934	0.4039	0.4332	0.4687	0.2305	0.2529	0.2865	0.1426	0.1610	0.1917
9	0.9403	0.9701	0.9933	0.4129	0.4388	0.4715	0.2412	0.2621	0.2937	0.1552	0.1725	0.1977
10	0.9440	0.9732	0.9951	0.4203	0.4430	0.4753	0.2476	0.2662	0.2952	0.1634	0.1778	0.2028

Critical values of ZN_k for Exponential samples with =5 (Zerbet and Nikulun, 2003,) for α =10%, α =5% and α =1%

n	k =2			k =3			k =4			k =5		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
6	5.46 51	10.6153	56.6260	1.3808	1.8114	3.5097						
7	5.3252	10.2597	53.2620	1.3915	1.8249	4.0296						
8	5.3882	10.4825	58.8740	1.3793	1.8163	3.6760	0.7805	0.9197	1.3961			
9	5.6169	10.1242	48.9738	1.3687	1.8230	3.8234	0.7856	0.9256	1.3863			
10	5.6551	10.5232	52.5709	1.3723	1.8319	3.7148	0.7898	0.9391	1.3490	0.5360	0.6112	0.8300

Critical values of SP_k for Exponential samples with =5 (Shadrokh and Pazira,2010) for α =10%, α =5% and α =1%